Differential Equations **Review Sheet** **Spring 2004**

1. Solve the DE

$$\frac{dy}{dx} = \frac{\sec^2 y}{1+x^2}$$

2. Solve the following boundary value problem

$$(1/x + 2y^2 x)dx + (2yx^2 - \cos y)dy = 0; y(1) = \pi$$

3. Solve the following boundary value problem

$$\frac{dy}{dx} - \frac{y}{x} = xe^x, y(1) = e - 1$$

4. Solve the differential equation

$$\frac{dy}{dx} - y = e^{2x} y^3$$

5. Solve the differential equation

$$(xy + y^2 + x^2)dx - x^2 dy = 0.$$

6. A brine solution of salt flows at a constant rate of 4 L/min into a large tank that initially held 100L of pure water. The solution inside the tank is kept well stirred and flows out of the tank at a rate of 2L/min. If the concentration of salt in the brine entering the tank is 0.2 kg/L, determine the mass of salt in the tank after *t* minutes. When will the concentration of salt in the tank reach 0.1 kg/L?

7. A cold beer initially at 35 degrees F warms up to 40 degrees F in 3 minutes while sitting in a room of temperature 70 degrees F. How warm will the beer be if it is left out for 20 minutes?

8. A garage with no heating or cooling has a time constant of 2 hr. If the outside temperature varies as a sine wave with a minimum of 50 degrees F at 2:00 A.M. and a maximum of 80

degrees F at 2:00 P.M., determine the times at which the building reaches its lowest temperature and its highest temperature, assuming the exponential term has died off.

Hint: Use the equations for the outside temperature $M(t) = M_0 - B\cos\omega t$ where M_0 represents the average outside temperature, and B is a positive constant; and for the inside temperature

$$T(t) = B_0 - BF(t) + Ce^{-kt}, \text{ where } F(t) = \left[1 + \left(\omega/K\right)^2\right]^{-\frac{1}{2}} \cos(\omega t - \varphi)$$

Bonus (+5) Show that the expressions

$$F(t) = \left[1 + \left(\omega/K\right)^2\right]^{-\frac{1}{2}} \cos(\omega t - \varphi)$$

, and

$$F(t) = \frac{\cos\omega t + \left(\omega/K\right)\sin\omega t}{1 + \left(\omega/K\right)^2}, \text{ where } \tan\varphi = \frac{\omega}{K}$$

are equivalent.