Test 4 **Spring 2019**

Complete problems 1 and 2 and 8 problems in the range 3-15. Extra problems done correctly, are worth a maximum total of 2 bonus points each.

1. $\int_1^\infty \frac{dx}{x^p}$ (Integrate, and find the limit for p >1 and for p = 1. Then, use inequalities to show p < 1.)

2. Find enough nonzero terms for the McClaurin series for y = cos(x), evaluated at 1, so that it is accurate to 17, as given by MathCAD. Write out all 17 digits. Write out all the terms of the cosine series you used.

Find the limits below.

3. $\lim_{x\to 0} \frac{e^x - (1-x)}{x}$

4. $\mathrm{y} = \lim_{x\to 1^+} (\ln x)^{x-1}$

5. $\lim_{x\to\infty} \frac{x}{\sqrt{x^2+1}}$

6. $\lim_{x\to 0^+} x^{1/x}$

7. $\lim_{x\to 0^+} (-x \ln x)$

Determine whether the series below converge or diverge using one of the following tests:
(a) correctly stating that it is a convergent or a divergent p-series (identifying p < 1 or p > 1),
(b) by limit comparing it to a convergent or divergent p-series. (limit comparison test)
(c) by using the ratio test to determine whether it converges or diverges
(d) by using the divergence test ($\lim_{n\to\infty} a_n$ is not equal to zero).

8. $\sum_{n=1}^{\infty} \frac{1}{n^{22/23}}$

9. $\sum_{n=1}^{\infty} \frac{2n+66}{n^3+3n+18}$

10. $\sum_{n=1}^{\infty} \frac{2n+6}{n^2-4n+7}$

11. $\sum_{n=1}^{\infty} \frac{n}{\sqrt[3]{3n^7+n+6}}$

12. $\sum_{n=1}^{\infty} \frac{2n+6}{4n+7}$

13. $\sum_{n=1}^{\infty} \frac{n^3}{2^n}$

14. $\sum_{n=1}^{\infty} \frac{1}{3^n+3}$

15. $\sum_{n=1}^{\infty} \frac{n^n}{n!}$