Calculus I TEST IV Summer 2018

1.

a. Use the web software to find the approximate area and the error, for the area under the function $y = 6x^6 + 3x^3$, above the x–axis, on [0, 5], for $n = 30$, using Simpson's rule and the Trapezoid rule.

b. What happens to the error as more intervals are selected using the Trapezoid or Simpson's rule?

2.

a. Find the error estimate for the function $y = 6x^6 + 3x^3$, on [0, 5], for $n = 30$, using the Simpson's rule estimate, $\frac{K_4(b-a)^5}{180n^4}$, and the Trapezoid rule estimate, $\frac{K_2(b-a)^3}{12n^2}$, where K_4 and K_2 represent, $\max_{x\in[0,5]} f^{(iv)}(x)$ and $\max_{x\in[0,5]} f''(x)$ respectively.

b. Do the answers to question 2.a. fall within the predicted range? Explain.

3. a. Rank the approximating rules Upper or Lower Rectangles, Trapezoid, and Simpson in order of their accuracy in estimating the error under polynomial curves.

b. As the power on the function increases, what will happen to the error and the percentage error using the Trapezoid rule or Simpson's rule?

c. Explain your answer to part b. above.

4. a. Explain how the Riemann rectangles are chosen to estimate the area under a curve.

b. $\lim_{\max \Delta x_i \to 0} \sum_{i=1}^{n} f(x_i^*)\Delta x_i$ equals the _____ _____ under the function ______ on [a, b] when _____ .

5. a. If $f(x) < 0$ on [a, b], $\int_a^b f(x)dx$ is ________. Explain why using this is true by indicating the signs of the elements inside summation formula $\lim_{\max \Delta x_i \to 0} \sum_{i=1}^{n} f(x_i^*)\Delta x_i$.

b. If $f(x)$ is both above and below the x–axis on [a, b], draw the three cases for the value of $\int_a^b f(x)dx$.

6. Using the **limit of Riemann sums**, find the area under the curve $y = 3x^2 - 2x + 3$ on [0, 3].

7. a. $\int \frac{t^4 + t^3 + t + 5}{\sqrt[3]{t^5}} dt$

b. $\int (x^2 + 2x + 6)^4 (x + 1) dx$

8. Express $\int_c^{c+h} f(x)dx - \int_c^h f(x)dx$ as one integral.

9. Answer the questions below about the steps in the proof of the first fundamental theorem of integral calculus:

If $f(x)$ differentiable on [a, b], then $\int_a^b f(x)\,dx = F(b) - F(a)$, where $F'(x) = f(x)$.

a. Why is it "legal" to rewrite the $F(b) - F(a)$ as a telescoping sum as below

$$F(b) - F(a) = \left[F(x_n) - F(x_{n-1})\right] + \left[F(x_{n-1}) - F(x_{n-2})\right] + \left[F(x_{n-2}) - F(x_{n-3})\right] \cdots + \left[F(x_1) - F(x_0)\right]$$

b. What allows you to rewrite each of the differences $[F(x_n) - F(x_{n-1})]$, … , $[F(x_1) - F(x_0)]$ as

$$F(x_i) - F(x_{i-1}) = F'(x_i^*)(x_i - x_{i-1})$$

c. Why is $(x_i - x_{i-1})$ equal to Δx_i?

d. Why is $F'(x_n^*)\Delta x_n + F'(x_{n-1}^*)\Delta x_{n-1} + \cdots + F'(x_1^*)\Delta x_1 = f(x_n^*)\Delta x_n + f(x_{n-1}^*)\Delta x_{n-1} + \cdots + f(x_1^*)\Delta x_1$?

e. Why is $f(x_n^*)\Delta x_n + f(x_{n-1}^*)\Delta x_{n-1} + \cdots + f(x_1^*)\Delta x_1 = \sum_{i=1}^{n} f(x_i^*)\Delta x_i$?

f. Why does $F(b) - F(a) = \sum_{i=1}^{n} f(x_i^*)\Delta x_i$ hold as $n \to \infty$?

10. Find $F'(x)$ for $F(x)=\int_{x^4}^{2}\left(5t+\frac{1}{t}\right)dt$

11. Use a picture to prove the rectangle rule for integrals: $m(b-a)\le\int_a^b f(x)dx \le M(b-a)$, where m and M are, respectively, the maximum and minimum values of $f(x)$ on $[a, b]$.

12. Prove the sum properties for definite integrals: $\int_a^b f(x)dx=\int_a^c f(x)dx+\int_c^b f(x)dx$ by showing both cases: (1) where c is in the interval [a, b] and (2) where c is outside of the interval [a, b].