

To receive partial credit you must show your work.

1. What is  $\lim_{x \rightarrow 3} \frac{x^4 - 81}{x^6 - 729}$  ?

2. What is  $\lim_{h \rightarrow 0} \frac{1}{h} \left[ \frac{1}{\sqrt{25+h}} - \frac{1}{5} \right]$  ?

3. Find the following limits, if they exist. If they do not exist, explain why they do not exist.

a.  $\lim_{x \rightarrow 8} \frac{3x}{(x+8)^2}$

b.  $\lim_{x \rightarrow 2^+} \frac{33}{x^2 - 3x - 10}$

c.  $\lim_{x \rightarrow 4^+} \frac{5}{4-x}$

4. Find  $\lim_{x \rightarrow -\infty} \frac{4-3x}{\sqrt{4+6x+4x^2}}$  .

6. Where, if anywhere, the function below is discontinuous? (To receive full credit, you must justify your answer by explicitly showing which part (s) of the definition of continuity fails to be true. )

$$f(x) = \begin{cases} \frac{|x-5|}{x-5} & , x \neq 5 \\ 1 & , x = 5 \end{cases}$$

7. Let  $f(x) = \begin{cases} cx^2 & , x < 1 \\ 3cx - 1 & , x \geq 1 \end{cases}$ . Determine all values of "c" so that f (x) is continuous for all real numbers.

8. State the definition of continuity for  $y = f(x)$  at  $x = a$ . Give both the formal definition and an *everyday* language description of each part of the definition.

9. State five definitions for the derivative.

10. Discuss the accuracy of secant slopes and tangent slopes. Can secant slopes be made more accurate? Explain. Can tangent slopes be made more accurate? Explain.

11. If the height of an object is given by the equation  $h(t) = 6t^2 - 100t + 500$ , where  $h$  is in feet and  $t$  is in seconds

a. What is the average velocity of the object over the interval  $[10, 12]$ ? Is this velocity a secant or a tangent?

b. What is the average velocity of the object over the interval  $[10, 10.1]$ ? Is this velocity a secant or a tangent?

c. Which of the above average velocities is closer to the instantaneous velocity at  $t = 10$ s? Why is it closer?

d. What is the instantaneous velocity at time  $t = 10$ s?

12. **Using the limit definition of the derivative** find the derivative of  $y = x^8$ .

13. Find the derivative of  $f(x) = (x^6 + 44x^2 + 16x)(x^5 - 13x + 16x^{-1})$

14. Prove the power rule.

15. Prove the product rule.

16. Find the derivative of  $H(x) = \frac{(\sqrt{x} + 3)(x - x^2)}{x^3 + 6x + 15}$ .