Module 8: LIMITS--A GRAPHICAL APPROACH

Exploring Calculus

In this discovery section you will learn about **limits,** one of the most important concepts in calculus. It distinguishes calculus from all previous mathematics courses. In calculus, we often need to know the answer to the question: If the input to a function is getting closer and closer to some x-value, to what y-value is the *output* getting closer? When we determine the answer to this question, we have found a limit.

Directions for running the Limit_ app.

- Sort the folder c:\temp\solve by type, by clicking on the header "Type".
- Scroll to the section of the folder that has "Windows Batch Files", and find the file named, Limit_.
- Double click on it to open it.
- Two windows will open. In the second window, there is a prompt.
- Type **N** and hit "enter"
- Respond to the question about both left and right limits by leaving the default response *left limit, 1* and hitting "enter"
- Leave the other values at their defaults, hitting "enter" at each step: increment size of **0.05,** x-value of **2,** and function $\mathbf{y = x^2}$.

*Watch carefully as the graph plots a series of points with x-values getting closer and closer to x = 2 -- from the left. More importantly watch as it shows the **y-values** approaching some number. This y-value resulting from x getting closer and closer to 2 is **the limit** as x approaches 2 from the left for the function* $y = x^2$.

Type in the to the nearest 10^{th}, to the questions Left Limit = ____ You will know your response is correct, when you see, "Good, Correct Response".

This is called the **left limit** as **x** approaches **2** for the function, written as fill in the blank): $\lim_{x \to 2^-}$ ______

Now, you have found a limit, a left-side limit. Let's find a right-side limit.

To re-run the Limit_ app:

- Click on the graph and hit "enter".
- Hit "enter" again to say "Y" you want to continue with the concept.
- Respond "N" to the question whether you want both left and right limits
- Then, type "0" (right limit), followed by "enter"
- Hit "enter" at the defaults: increment size 0.05, x-value of 2, and function y = x^3.

The graph will paint a series of points and then ask the question, Right Limit = _____.

Fill in blank for the **right limit** you have found, $\lim_{x \to 2^+} =$ _____

The left and right limits you found above are called *one-sided limits*.

If at some x-value **a,** the left-side and the right-side limit are the same, their common **y-value** called **L,** is **the limit** of that function at **a**. Mathematically, this is written

If $\lim_{x \to a^-} f(x) = \lim_{x \to a^-+} f(x) = L,$ then $\lim_{x \to a^-} f(x) = \lim_{x \to a^-+} f(x) = \lim_{x \to a} f(x) = L$

Put another way, for “the” the limit of a function to exist, the left and right limits must exist and be equal.

Go back and re-run left-side and right-side limits for f(x) = x^2, at x – 2. Notice that the left and right limits for the function y = x^2 are ____ and that we may therefore say the unsigned limit, $\lim_{x \to 2} x^2 =$ _____ .

When “the” limit does not exist.

There are two cases where we say the limit at some x-value **a** does not exist.

case 1

Both $\lim_{x \to a^-} f(x)$ and $\lim_{x \to a^-+} f(x)$ *exist,* but are *not* equal.

case 2 We use $\pm\infty$ as a limit to designate that at some x-value the curve has **blown-up** in the positive or negative direction. Technically, $\pm\infty$ are not limits, because the graph does not approach a certain y-value; however, they are useful for specifying how a curve has blown up.

So, if either $\lim_{x \to a^-} f(x) = \pm\infty$ or $\lim_{x \to a^+} f(x) = \pm\infty$ we say the limit as **x** approaches **a** from the left or the right equals $\pm\infty$.

If both the left and the right limit at some x-value "a" are $+\infty$, or they are both $-\infty$, then we say that the limit as x approaches "a" is either $+\infty$ or $-\infty$, whichever is correct.

Critical Thinking Questions

1. Now consider the function $f(x) = \begin{cases} 6 - x^2, x \leq 2 \\ x + 2, x > 2 \end{cases}$.

This is called a function with a *split domain*.

a1. $\lim_{x \to 2^-} =$ _____

a2. $\lim_{x \to 2^+} =$ _____

b. Does the limit at x = 2 exist?

c. Why or why not?

2. Now choose both limits at x = 0 , for $f(x) = \begin{cases} x^2, x < 0 \\ x + 1, x \geq 0 \end{cases}$

a. $\lim_{x \to 0^-} =$ _____

b. $\lim_{x \to 0^+} =$ _____

c. Does the limit at x = 0 exist?

d. Why or why not?

3. Finally select both limits and choose $f(x) = 1/x$ for both the left and right functions, at x = 0.

a. $\lim_{x \to 0^-} =$ _____

b. $\lim_{x \to 0^+} =$ _____

c. $\lim_{x \to 0} =$ _____

Skill Exercises

For problems 1-10 , find the limits. If any do not exist write *DNE* . If any are or , indicate so.

Be sure to check the left and right limits when necessary.

1. $\lim_{x\to 3}(2x-7)$ = _____

2. $\lim_{x\to 1}\frac{(x+1)}{(x+2)}$ = _____

(In Limit_ app, change the increment size to 0.01 and be sure to enclose the numerator and denominator in parentheses.)

3 $\lim_{x\to 0}\frac{1}{(x^2)}$ = _____

4. $\lim_{x\to -2} f(x)=$ _____ where $f(x)=\begin{cases}3-x, x\le -2\\ 5, x>-2\end{cases}$

5. $\lim_{x\to 0} g(x)$ = _____ where $g(x)=\begin{cases}x^2-4, x<0\\ 4-x^2, x\ge 0\end{cases}$

6. $\lim_{x\to 3}\frac{(x^2-9)}{(x-3)}$ = _____ (Be sure to enclose the numerator and denominator in parentheses in Limit_ app.)

7. $\lim_{x\to 2^-}\frac{1}{(x-2)}$ = _____

8. $\lim_{x \to 2^+} \left(1 + sqrt(x-2)\right) =$ _____

9. $\lim_{x \to \pi} sin(x) =$ _____ (Enter in Limit_ as pi.)

10. $\lim_{x \to 4} \frac{\left(sqrt(x) - 2\right)}{(x-4)} =$ _____

(Be sure to enclose the numerator and denominator in parentheses in Limit_ app.)

For problems 11-14, be sure to use *proper limit notation.*

For problems 11-14, be sure to state the *x-value* and the *limit* for each function.

11. Find *two examples* of functions where the limit at some x-value exists.

12. a. Write a function with ***split domain*** where the limit does not exist at x = 2 .

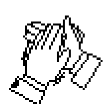
b. What is the left limit?

c. What is the right limit?

13. a. Find an example of a function where the left limit is $+\infty$ and the right limit is $-\infty$.

b. What is the limit at that x-value?

14. Find an example of a function where the limit is $-\infty$.



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