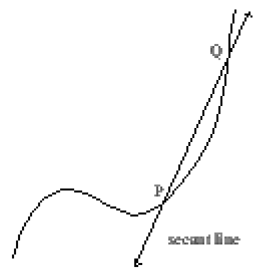
Module 10: SECANT AND TANGENT LINES

Exploring Calculus

Everyone can picture a line tangent to a circle. In calculus we consider lines tangent to a curve, and use them to define the **slope** of a curve. Knowing the slope of a curve will allow us to solve many problems which we otherwise could not. As you will see, this concept has many real world applications.

If we draw a curve which is not a line, and construct a line which passes through the curve in two points, **P** and **Q**, then that line is called a **secant line**. We can easily calculate the slope of this line since we know the points P and Q.



If we move the above secant line so that it only crosses the curve at point P, it is then called a **tangent** line. Finding the slope of this line is the subject of this module.

Critical Thinking Questions

1. Draw an *accurate* graph of $y = x^2$ over the interval [-3, 3].

   a. Draw the secant line to $y = x^2$ through (0, 0) and (2, 4). What is the slope of this line?

b. Using the same graph, draw the secant line to $y = x^2$ through (0, 0) and (1, 1). What is the slope of this line?

c. Which of the two secant lines above has slope equal to the slope of the tangent to the curve $y = x^2$ at (0, 0)?

d. Does the slope of the secant line through $(0, 0)$ and $(1/2, 1/4)$ equal the slope of the line tangent to the curve $y = x^2$ at (0, 0)?

e. What should you do to the second point above so that the slope of the secant line more closely approximates the slope of the line tangent to the curve at (0, 0) ?

f. What is the slope of the line tangent to $y = x^2$ at (0, 0)?

Display the graph. **(Tangent_.bat w/ default values.)**

2. Draw a second graph of $y = x^2$ below, with four secant lines so that each goes through the graph at x = 1 and the other points on the graph are at the x-values $4, 2, 3/2,$ and $5/4$.

Label the points.

a. Algebraically determine the slope of each of these secant lines.

b. What is the slope of the line tangent to $y = x^2$ at x = 1 ?

Display the graph to verify your answer to Q2b. above. **(Tangent_.bat, left = 0, right = 2, x = 1.)**

3. a. In the problems above, you have seen that the slope of the secant line gets _____and _____ to the slope of the _____ line as the _____- values get _____ together.

b. The phrase *closer and closer* suggests the mathematical term _____.

4. Let P(a, f (a)) be a fixed point on the curve y = f (x) and let Q(x, f (x)), where x a, be a neighboring point on the same curve.

Let $\mathbf{m}_{SEC}$ represent the slope of the secant line through the points P(a, f (a)) and Q(x, f (x)).

Let $\mathbf{m}_{TAN}$ represent the slope of the tangent line to y = f (x) at the point P(a, f (a)).

Fill in the blanks below using the correct mathematical symbols.

a. $\mathbf{m}_{SEC} = \dfrac{f(x) - \_}{x - \_}$

Use the slope formula with the points P and Q for part a.

b. $\mathbf{m}_{TAN} = \lim_{\_ \to \_} \frac{f(x) - \_}{x - \_}$

5. Use  to help determine the answers to the questions below. **(Tangent_.bat, left = x-value - 1, right = x-value + 1.)**

When using the graph below, choose the left and right endpoint one (1) unit on either side of the point of tangency.

a. For the function y = x , find the slope of the tangent line at

x = -1 $\mathbf{m}_{TAN}$ = ____________

x = 0 $\mathbf{m}_{TAN}$ = ____________

x = 2 $\mathbf{m}_{TAN}$= ____________

b. For the function $y = x^2$, find the slope of the tangent line at

x = -1 $\mathbf{m}_{TAN}$= ____________

x = 0 $\mathbf{m}_{TAN}$ = ____________

x = 2 $\mathbf{m}_{TAN}$ = ____________

c. For the function $y = x^{3,}$ find the slope of the tangent line at

x = -1 $\mathbf{m}_{TAN}$ = ____________

x = 0 $\mathbf{m}_{TAN}$ = ____________

x = 2 $\mathbf{m}_{TAN}$ = ____________

6. a. The slope of a line _____ _____ _____, as the x-value changes.

b. We can *approximate* the *slope* of a curve using _____ lines.

c. The slope of the tangent to a curve _____as the x-value changes.

d. The approximations to the slope of a curve become better and better as the second x-value_____ the first x-value.

e. We call **m** **TAN** *the slope* of the curve because it is the _____ of the slope of the _____ lines, which _____ the slope of the curve _____ and _____ as the _-values get ____ and _____ together.

7. The average rate of change of **y** with respect to **x** (denoted $\Delta y/\Delta x$) over the interval [x, x + h] is defined as the **difference quotient** $[f(x+h)-f(x)]/h$.

a. Find the difference quotient for the function f (x) = x^2.

b. What is the limit as h → 0 zero for the above difference quotient?

c. The function you found in Q.7b is called the derivative of f (x) --- written as **f '(x)**.

What is f '(2)?

d. How could you use  to verify your answer to Q.7c? Is your answer to Q.7c correct?

Skill Exercises

1. Consider the quadratic function f (x) = x^2 - 4x - 5.

a. Sketch the graph of f (x) by finding its vertex and intercepts.

b. Use  to find **m** **TAN** at x = 1. **(Tangent_.bat, left = 0, right = 2, x = 1, change to y = x^2 - 4x - 5.)**

c. Find the equation of this tangent line and write your answer in slope-intercept form.

d. Graph the tangent line and label the point of tangency.

2. Given the function f (x) = 12 + $2x^2$ - x^4.

a. Find the slopes of the tangent lines to the graph of f (x) at the x-values -1, 0 and 1.

x = -1 **m** **TAN**= ____________

x = 0 **m** **TAN**= ____________

x = 1 **m** **TAN**= ____________

b. What relationship do you notice between the slopes of the tangent lines and the behavior of the function at these points?

c. Find f (-1), f (0) and f (1).

3. For the function $f(x) = x^2 + 6$

a. Algebraically determine $\Delta y / \Delta x$ as **x** changes from **2** to **3**.

b. What is the slope of the tangent line to the curve at x = 2?

c. What is f '(2)?

d. Compare the values in Q.3a--c, and explain why they are related as they are.

4. Suppose that the distance **D** (in feet) covered by a car moving along a straight road **t** seconds after starting from rest is given by the function

$D(t) = 2t^2 + 48t$

a. Calculate the **average velocity** $\Delta D / \Delta t$ over the time intervals

20 to 21 seconds _____

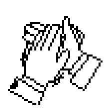
20 to 20.01 seconds _____ (Express your answer to the nearest hundredth)

b. The average velocities above get closer and closer to the slope of the graph of D (t), at t = _____ sec.

c. How fast is the car traveling at the *instant* t = 20 seconds?

d. What is the relationship between $D'(t)$ and $\mathbf{m}_{TAN}$ at t = 20.

e. D(t) represents the __________covered by the car at any time t, and $D'(t)$ represents__________ __________ of the car at any time t.



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